

PHASE-DIRECT CHANNEL ESTIMATION FOR SPACE-TIME OFDM

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ABSTRACT

Space time (ST) OFDM can achieve high data rate transmissions with diversity gains. However, channel estimation is necessary at the receiver end to recover the signal. To further improve the subspace-based estimated channel, we introduce phase direct (PD) in the ST OFDM system. PD is originally used in OFDM system, and not suited in ST OFDM. Hence, we exploit the transmitted data's spatial and temporal correlation to develop a sum-difference square algorithm to solve this problem for ST OFDM.

1. INTRODUCTION

In OFDM the entire channel is divided into many narrow parallel subchannels, thereby increasing the symbol duration and reducing the intersymbol interference (ISI) caused by the multipath environments. Moreover, Space-time (ST) codes [1] realize the diversity gains by introducing temporal and spatial correlation into the signals transmitted from different antennas without increasing the total transmitted power or transmission bandwidth.

Transmitter diversity is an effective technique for combating fading in mobile wireless communications, especially when receiver diversity is expensive or impractical. Many researchers [2,3] have studied transmitter diversity for wireless systems. We focus on two transmit-antennas and one receive-antenna and use the well known Alamouti's block ST code [4].

For most ST code transceivers, multichannel estimation algorithms are needed. [5] had shown a training-based estimation of frequency-selective channels in ST-OFDM. However, training sequences consume bandwidth and incur spectral efficiency (and thus capacity) loss. For this reason, blind channel estimation methods, such as a subspace based algorithm [6], receive growing attention for block precoded ST-OFDM.

To further improve the channel estimation, the finite alphabet property can be exploited to better the subspace-based channel estimates by applying the "phase direct (PD)" methods. Its main idea is to solve the phase ambiguities after we get the channel power response. For conventional OFDM system, it is very easy to get the channel power response. But in ST OFDM, it is quite a different case, since the received data is composed of two different transmitted data; it's not easy to separate them. The main problem we face now is how to get the channel power response, which is hard to get in general. Hence, we only focus on BPSK system and exploit the special characteristic of ST OFDM to develop a sum-difference square algorithm to solve this problem.

The rest of this paper is organized as follows. After presenting the system model in Section 2, we show PD method in Section 3

and decision-direct (DD) method in Section 4. Section 5 presents simulation results, and section 6 gathers our conclusions.

2. SPACE TIME OFDM SYSTEM MODEL

Fig. 1 depicts the ST OFDM system with two transmit antennas and one receive-antenna. Prior to transmission, the data symbols $\underline{s}(n)$ are first grouped into super blocks of size $2K \times 1$,

$$\underline{s}(n) = \begin{bmatrix} \underline{s}^{(1)}(n) \\ \underline{s}^{(2)}(n) \end{bmatrix} \quad (1)$$

where we indicate the first K symbols as $\underline{s}^{(1)}(n)$ and last K symbols as $\underline{s}^{(2)}(n)$. $\underline{s}(n)$ is fed to the ST encoder. The encoder takes input two consecutive blocks $\underline{\bar{s}}^{(1)}(n)$ and $\underline{\bar{s}}^{(2)}(n)$ to output the following $2M \times 2$ code matrix:

$$\begin{bmatrix} \underline{\bar{s}}_1(n) & \underline{\bar{s}}_2(n) \end{bmatrix} = \begin{bmatrix} \underline{\bar{s}}_1^{(1)}(n) & \underline{\bar{s}}_2^{(1)}(n) \\ \underline{\bar{s}}_1^{(2)}(n) & \underline{\bar{s}}_2^{(2)}(n) \end{bmatrix} = \begin{bmatrix} \underline{\bar{s}}^{(1)}(n) & \underline{\bar{s}}^{(2)}(n) \\ -\underline{\bar{s}}^{(2)*}(n) & \underline{\bar{s}}^{(1)*}(n) \end{bmatrix} \quad (2)$$

where

$$\underline{\bar{s}}_1(n) = \begin{bmatrix} \underline{\bar{s}}_1^{(1)}(n) \\ \underline{\bar{s}}_1^{(2)}(n) \end{bmatrix} \quad \text{and} \quad \underline{\bar{s}}_2(n) = \begin{bmatrix} \underline{\bar{s}}_2^{(1)}(n) \\ \underline{\bar{s}}_2^{(2)}(n) \end{bmatrix} \quad (3)$$

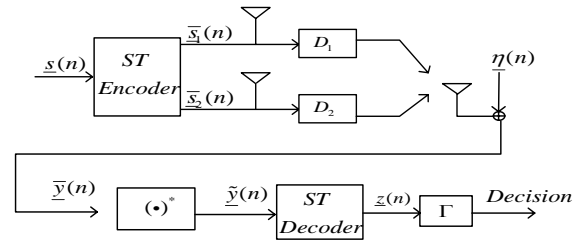


Fig.1 ST-OFDM transceiver

We assume in what follows that the channels between the two transmit antennas and the receive antenna are frequency selective and that their baseband equivalent effect in discrete time is captured by an FIR linear time-invariant filter with impulse response vector $\underline{h}_i = [h_i(0), \dots, h_i(L)]$ $i=1,2$, where L is an upper bound for the channel orders of \underline{h}_1 and \underline{h}_2 , i.e., $L \geq \max(L_1, L_2)$ with L_i being the channel order for \underline{h}_i . Let $D(H_1)$ and $D(H_2)$ be the diagonal matrices corresponding to subchannel

$D(H_i) = \text{diag}(H_i(0), H_i(1), \dots, H_i(M-1))$, and $H_i(\rho) = \sum_{l=0}^L h_i(l)\rho^{-l}$ is the frequency response of channel $h(l)$ at point ρ . Moreover, we can write

$$\underline{H}_i = \mathbf{V} \underline{h}_i = [H_i(e^{j0}), \dots, H_i(e^{j2\pi(M-1)/M})]^T \quad (4)$$

with \mathbf{V} denoting submatrix consisting of the first $L+1$ columns of the FFT matrix. From Fig.1, we can have

$$\begin{aligned} \underline{\tilde{y}}(n) &= \mathbf{D}(\underline{H}_1) \underline{\tilde{s}}(n) + \mathbf{D}(\underline{H}_2) \underline{\tilde{s}}(n) + \underline{v}(n) \\ &= \begin{bmatrix} \mathbf{D}(\underline{H}_1) \underline{\tilde{s}}^{(1)}(n) + \mathbf{D}(\underline{H}_2) \underline{\tilde{s}}^{(2)}(n) \\ -\mathbf{D}(\underline{H}_1) \underline{\tilde{s}}^{(2)*}(n) + \mathbf{D}(\underline{H}_2) \underline{\tilde{s}}^{(1)*}(n) \end{bmatrix} + \begin{bmatrix} \underline{v}^{(1)}(n) \\ \underline{v}^{(2)*}(n) \end{bmatrix} \end{aligned} \quad (5)$$

Then,

$$\begin{aligned} \underline{\tilde{y}}(n) &= \begin{bmatrix} \underline{\tilde{y}}^{(1)}(n) \\ \underline{\tilde{y}}^{(2)*}(n) \end{bmatrix} = \begin{bmatrix} \mathbf{D}(\underline{H}_1) & \mathbf{D}(\underline{H}_2) \\ \mathbf{D}(\underline{H}_2^*) & -\mathbf{D}(\underline{H}_1^*) \end{bmatrix} \begin{bmatrix} \underline{\tilde{s}}^{(1)}(n) \\ \underline{\tilde{s}}^{(2)}(n) \end{bmatrix} + \begin{bmatrix} \underline{v}^{(1)}(n) \\ \underline{v}^{(2)*}(n) \end{bmatrix} \\ &= \underline{\mathbf{D}} \underline{s}(n) + \underline{\eta}(n) \end{aligned} \quad (6)$$

where \mathbf{H} , $\underline{s}(n)$ and $\underline{\eta}(n)$ are defined, respectively, as

$$\underline{\mathbf{D}} = \begin{bmatrix} \mathbf{D}(\underline{H}_1) & \mathbf{D}(\underline{H}_2) \\ \mathbf{D}(\underline{H}_2^*) & -\mathbf{D}(\underline{H}_1^*) \end{bmatrix}, \quad \underline{\eta}(n) = \begin{bmatrix} \underline{v}^{(1)}(n) \\ \underline{v}^{(2)*}(n) \end{bmatrix} \quad (7)$$

When the channel matrices \underline{H}_1 and \underline{H}_2 become available at the receiver, it is possible to demodulate $\underline{\tilde{y}}(n)$ with diversity gains by a simple matrix multiplication

$$\begin{aligned} \underline{z}(n) &= \underline{\mathbf{D}}^H \underline{\tilde{y}}(n) \\ &= \begin{bmatrix} \mathbf{D}(\underline{H}_1^*) & \mathbf{D}(\underline{H}_2) \\ \mathbf{D}(\underline{H}_2^*) & -\mathbf{D}(\underline{H}_1) \end{bmatrix} \begin{bmatrix} \mathbf{D}(\underline{H}_1) & \mathbf{D}(\underline{H}_2) \\ \mathbf{D}(\underline{H}_2^*) & -\mathbf{D}(\underline{H}_1^*) \end{bmatrix} \underline{s}(n) + \underline{\mathbf{D}}^H \underline{\eta}(n) \\ &= \begin{bmatrix} \mathbf{D}_{12} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{12} \end{bmatrix} \underline{s}(n) + \underline{\xi}(n) \end{aligned} \quad (8)$$

where

$$\mathbf{D}_{12} = \mathbf{D}(\underline{H}_1^*) \mathbf{D}(\underline{H}_1) + \mathbf{D}(\underline{H}_2^*) \mathbf{D}(\underline{H}_2) \quad (9)$$

$$\underline{\xi}(n) = \underline{\mathbf{D}}^H \underline{\eta}(n) \quad (10)$$

The soft decision data can be computed by a simple division, and projected onto the finite alphabet to get the hard decision $\underline{\hat{s}}(n)$.

However the key to solve (8) is to get the channel information $\mathbf{D}(\underline{H}_1)$ and $\mathbf{D}(\underline{H}_2)$. Here we use the subspace based method in [6] to estimate the channel and use PD to improve channel estimation.

3. PHASE-DIRECT CHANNEL ESTIMATION

Before addressing PD [7] to ST OFDM, we first show how it works in conventional OFDM. The data for the m th subcarrier on the n th received data block is written as

$$y(n, m) = H(\rho_m) s(n, m) + \eta(n, m) \quad (11)$$

where $s(n, m)$ is the transmitted data corresponding to the m th subcarrier on the n th block, $H(\rho_m)$ is the frequency channel of the m th subcarrier, and $\eta(n, m)$ is the noise term.

Now, we focus on PSK constellation of size P : $\{\zeta_p = e^{j2\pi p/P}\}_{p=1}^P$.

We take the power of P to (11) and omit the noise to get

$$E\{y^P(n, m)\} = E\{[H(\rho_m) s(n, m)]^P\} = H^P(\rho_m) E\{s^P(n, m)\} \quad (12)$$

In practice, $E\{y(i, m)^P\}$ is replaced by its sample average:

$$H^P(\rho_m) = \frac{1}{N} \sum_{n=0}^{N-1} \frac{y^P(n, m)}{E\{s^P(n, m)\}} \quad (13)$$

where N is the received data block number.

Since we focus on PSK constellation of size P , we have $E\{s^P(n, m)\} = 1$ and (13) becomes

$$H^P(\rho_m) = \frac{1}{N} \sum_{n=0}^{N-1} y^P(n, m) \quad (14)$$

Hence, we have got the channel power response. Next, we only need to get the channel phase response, which means for each $m \in [0, M-1]$ (assuming a total of M subcarriers), we have

$$\hat{H}(\rho_m) = \lambda_m [H^P(\rho_m)]^{1/P} \quad (15)$$

where

$$\lambda_m \in \{e^{j(2\pi/P)p}, p = 1, \dots, P\} \quad (16)$$

is the corresponding phase ambiguity in taking the P th root.

For each $m \in [0, M-1]$, we can resolve the phase ambiguity by searching over candidate phase values

$$\lambda_m = \arg \min_{\lambda_m} \|H_{est}(\rho_m) - \lambda_m [H^P(\rho_m)]^{1/P}\|^2 \quad (17)$$

where $H_{est}(\rho_m)$ will be discussed later.

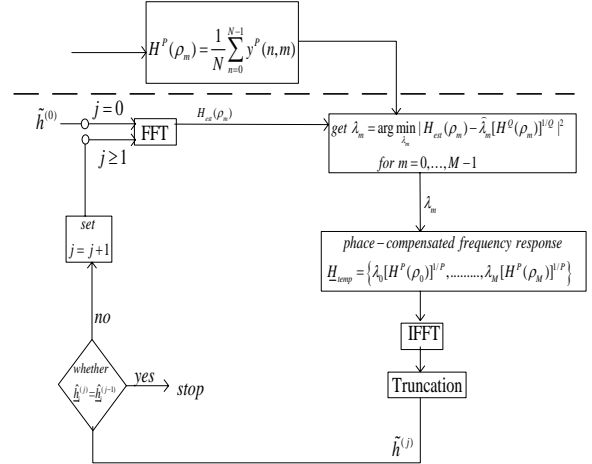


Fig.2 Signal-flow graph of phase direct in conventional OFDM

Therefore, we can improve channel estimation accuracy through the following Phase Directed (PD) steps shown in Fig. 2:

Step (1) Set $j=0$, find an initial estimate $\hat{h}_{(0)}$ using any estimation method, and compute the frequency response $\hat{H}_{(0)}$, then we can get $H_{est}(\rho_m) = \hat{H}_{(0)}(\rho_m)$ for $m=0, 1, 2, \dots, M-1$.

Step (2) In each successive iteration, j is added by 1, and

(a) Resolve phase ambiguities by replacing $H_{est}(\rho_m)$ with

$$H_{(j-1)}(\rho_m) \text{ in (17), and then form the vector}$$

$$\underline{H}_{temp} = \{\lambda_0 [H^P(\rho_0)]^{1/P}, \dots, \lambda_{M-1} [H^P(\rho_{M-1})]^{1/P}\} \quad (18)$$

(b) Update time domain channel estimates

$$\hat{\underline{h}}_{(j)} = \mathbf{V}^H \underline{H}_{temp} \quad (19)$$

and their frequency response using

$$\hat{H}_{(j)} = \mathbf{V} \hat{\underline{h}}_{(j)} \quad (20)$$

where \mathbf{V} is the first $L+1$ columns of the DFT matrix. Note that here (18) means to perform an M -point inverse DFT on \underline{H}_{temp} and truncate the output by keeping only the first $L+1$ entries (since we have assumed the channel delay spread is equal or less than L); (19) amounts to performing an M -point DFT on a vector formed after zero-padding, we call (18) and (19) denoising.

Step 3) Repeat Step2 several times, or continue until channel estimates $\hat{h}_{(j)}$ converges within some tolerance.

4. PD in ST-OFDM based on sum-difference square algorithm

Here, we want to apply PD to the ST OFDM system. As described earlier, the main idea of PD is to solve the phase ambiguities in (17) after we get the channel power response in (15). For conventional OFDM system, it is very easy to get the channel power response from (14), but here it is quite a different case, since the received data is composed of two different transmitted data in (6); which are not easy to separate. Now we will derive an algorithm to get the channel power response.

Let's start from (6) and focus on BPSK baseband system which means we only use ± 1 data because it's hard to solve the channel power response for other system. We will get

$$\underline{\tilde{y}}(n) = \begin{bmatrix} \mathbf{D}(\underline{H}_1)\underline{\tilde{s}}^{(1)}(n) + \mathbf{D}(\underline{H}_2)\underline{\tilde{s}}^{(2)}(n) \\ -\mathbf{D}(\underline{H}_1^*)\underline{\tilde{s}}^{(2)}(n) + \mathbf{D}(\underline{H}_2^*)\underline{\tilde{s}}^{(1)}(n) \end{bmatrix} + \begin{bmatrix} \underline{v}^{(1)}(n) \\ \underline{v}^{(2)*}(n) \end{bmatrix} \quad (21)$$

For simplicity, we only take the m th data part and omit the noise.

$$\begin{aligned} y_m^{(1)}(n) &= m\text{th data of } \underline{\tilde{y}}^{(1)}(n), y_m^{(2)}(n) = m\text{th data of } \underline{\tilde{y}}^{(2)}(n) \\ s_m^{(1)}(n) &= m\text{th data of } \underline{\tilde{s}}^{(1)}(n), s_m^{(2)}(n) = m\text{th data of } \underline{\tilde{s}}^{(2)}(n) \end{aligned} \quad (22)$$

Then we get

$$\begin{aligned} y_m^{(1)}(n) &= H_1(\rho_m)s_m^{(1)}(n) + H_2(\rho_m)s_m^{(2)}(n) \\ y_m^{(2)}(n) &= -H_1(\rho_m)s_m^{(2)}(n) + H_2(\rho_m)s_m^{(1)}(n) \end{aligned} \quad (23)$$

Our purpose is to get $H_1^2(\rho_m)$ and $H_2^2(\rho_m)$. Square (23) and note that $(s_m^{(1)}(n))^2 = (s_m^{(2)}(n))^2 = 1$ for BPSK

$$\begin{aligned} (y_m^{(1)}(n))^2 &= H_1^2(\rho_m) + H_2^2(\rho_m) + 2H_1(\rho_m)H_2(\rho_m)s_m^{(1)}(n)s_m^{(2)}(n) \\ (y_m^{(2)}(n))^2 &= H_1^2(\rho_m) + H_2^2(\rho_m) - 2H_1(\rho_m)H_2(\rho_m)s_m^{(1)}(n)s_m^{(2)}(n) \end{aligned} \quad (24)$$

Here we adopt BPSK since we can have $(s_m^{(1)}(n))^2 = (s_m^{(2)}(n))^2 = 1$. For other PSK constellation of size P , we have to take the power of P to (23) to make $(s_m^{(1)}(n))^P = (s_m^{(2)}(n))^P = 1$ and the computation becomes more complex.

By taking their sum and difference of (24), we have

$$(y_m^{(1)}(n))^2 + (y_m^{(2)}(n))^2 = 2(H_1^2(\rho_m) + H_2^2(\rho_m)) \quad (25a)$$

$$\begin{aligned} (y_m^{(1)}(n))^2 - (y_m^{(2)}(n))^2 &= 4H_1(\rho_m)H_2(\rho_m)s_m^{(1)}(n)s_m^{(2)}(n) \\ &= \pm 4H_1(\rho_m)H_2(\rho_m) \end{aligned} \quad (25b)$$

From (25b),

$$H_2(\rho_m) = \frac{(y_m^{(1)}(n))^2 - (y_m^{(2)}(n))^2}{\pm 4H_1(\rho_m)} \quad (26)$$

Squaring both sides of (26), we have

$$H_2^2(\rho_m) = \frac{[(y_m^{(1)}(n))^2 - (y_m^{(2)}(n))^2]^2}{16H_1^2(\rho_m)} \quad (27)$$

Substitute (27) to (25a),

$$\frac{(y_m^{(1)}(n))^2 + (y_m^{(2)}(n))^2}{2} = H_1^2(\rho_m) + \frac{[(y_m^{(1)}(n))^2 - (y_m^{(2)}(n))^2]^2}{16H_1^2(\rho_m)} \quad (28)$$

Then, we have

$$16H_1^4(\rho_m) - 8[(y_m^{(1)}(n))^2 + (y_m^{(2)}(n))^2]H_1^2(\rho_m) + [(y_m^{(1)}(n))^2 - (y_m^{(2)}(n))^2]^2 = 0$$

Finally,

$$H_1^2(\rho_m) = \frac{(y_m^{(1)}(n) - y_m^{(2)}(n))^2}{4} \quad \text{or} \quad \frac{(y_m^{(1)}(n) + y_m^{(2)}(n))^2}{4} \quad (29)$$

$H_1^2(\rho_m)$ can be known from the square of the sum or difference of the received data. Similarly we can get $H_2^2(\rho_m)$ from (29). Since the two channels we use are assumed to be different, we can know that if $H_1^2(\rho_m)$ is one of (29) then $H_2^2(\rho_m)$ is the other one.

How to know which one of (29) is $H_1^2(\rho_m)$? We only need to compare it with the estimated channel via the subspace-based method to see which one has smaller Euclidean distance.

$$H_1^2(\rho_m) = \arg \min_{H_1^2(\rho_m)} \left\| H_{est}^2(\rho_m) - \frac{[y_m^{(1)}(n) \pm y_m^{(2)}(n)]^2}{4} \right\|^2 \quad (30)$$

where $H_{est}(\rho_m)$ is the estimated channel via the subspace-based method. See Fig.3 for the signal-flow graph of finding $H_1^2(\rho_m)$ and $H_2^2(\rho_m)$.

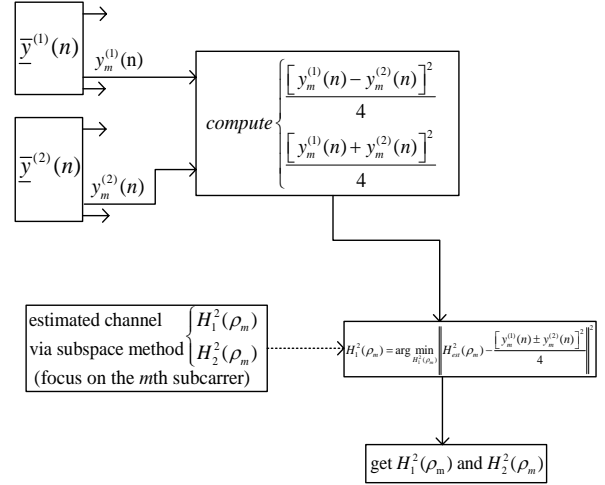


Fig.3 Sum-difference square algorithm to find $H_1^2(\rho_m)$ and $H_2^2(\rho_m)$

After obtaining $H_1^2(\rho_m)$ and $H_2^2(\rho_m)$, corresponding to the two channels' m th subcarrier, we still need to get all other subcarriers' frequency channel power

$$\underline{H}_i^2 = [H_i^2(\rho_0), H_i^2(\rho_1), \dots, H_i^2(\rho_{M-1})] \quad (31)$$

by using the same method as discussed above. Furthermore, in practice if we have received more than one data block (assume N), for every block n we need to get the frequency channel power $\underline{H}_{i,(n)}^2$ and then average all of them to get \underline{H}_i^2 .

$$\underline{H}_i^2 = \frac{1}{N} \sum_{n=0}^{N-1} \underline{H}_{i,(n)}^2 \quad (32)$$

After we've got \underline{H}_i^2 , simply apply to (17) by setting $P=2$ and follow the PD steps for further improvement. See Fig.4 for the signal-flow graph of PD on space-time OFDM in static channel.

5. COMPUTER SIMULATIONS

In our simulation, the subspace-based estimator [6] is used for channel initialization. Decision-direct (DD) in [8] is used for comparison. DD originally works in conventional OFDM, which only requires simple scalar division. Based on the ST data matrix in Section 2, we extend it to ST-OFDM, which corresponds to a matrix inverse and multiplication.

The performance for channel estimation is shown by the normalized mean-squares channel error (NMSCE) defined as:

$$\frac{\|h - \hat{h}\|^2}{\|\hat{h}\|^2} = \frac{\|\Delta h\|^2}{\|\hat{h}\|^2} \quad (33)$$

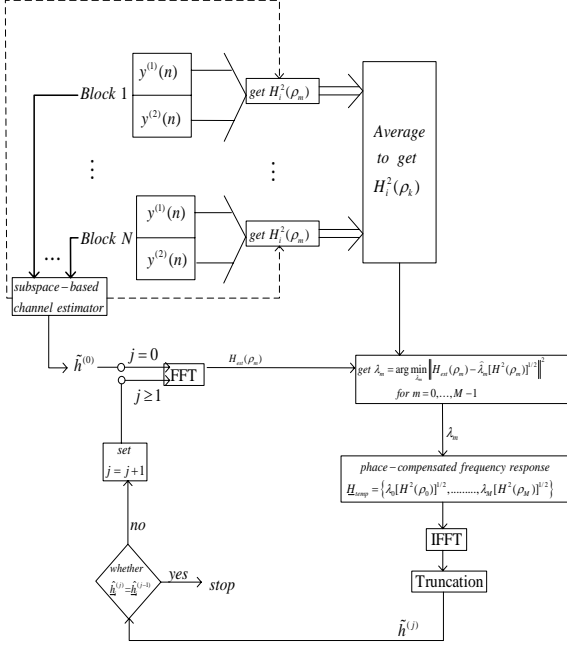


Fig.4 Signal-flow graph of phase direct on space-time OFDM

Here, we examine the estimator error as a function of the input SNR by using the following setup: A BPSK system with $L=4$ (five-ray channels) Rayleigh fading channel and $N=100$.

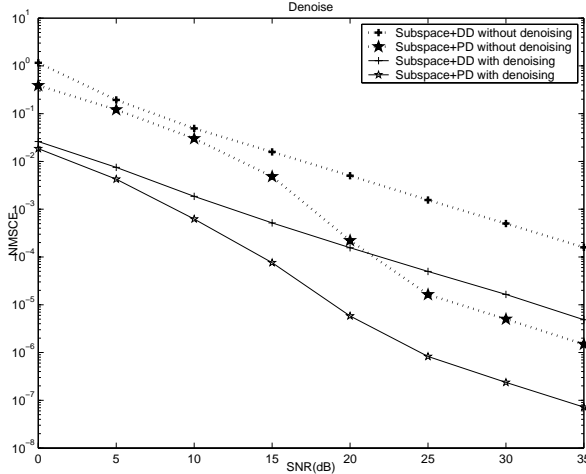


Fig.5. Channel errors of PD and DD with and w/o denoising

With denoising, the performance improves since we truncate the last $M-L$ point. From Fig.6, we can see that DD and PD with denoising better the subspace-based estimator. Moreover, PD improves significantly.

The main difference between PD and DD is that DD alternates between channel estimation and symbol detection while PD avoids symbol estimation by decoupling channel estimation from symbol recovery. Therefore, PD is immune to the well-known error propagation phenomenon that is present in DD iterations.

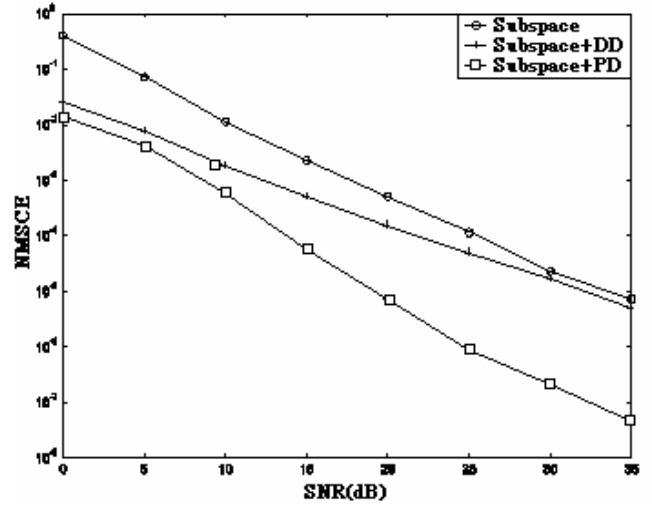


Fig.6 PD and DD v.s. subspace-based estimator

6. CONCLUSION

We have applied PD to the ST OFDM system, and see it really improves the performance. Moreover, we have compared both phase- and decision- direct methods and explain their main difference. For ST OFDM, the received data is composed of two different transmitted data, analysis is not easy, so we only focus on BPSK. Other systems such as QPSK, QAM need further study.

7. REFERENCES

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