COMMON-ACOUSTIC-POLES/ZEROS APPROXIMATION OF HEAD-RELATED TRANSFER FUNCTIONS

C. J. Liu and S. F. Hsieh

Department of Communication Engineering
National Chiao Tung University
Hsinchu, Taiwan 300
email:cjliu.cm89g@nctu.edu.tw and sfhsieh@cc.nctu.edu.tw

ABSTRACT

Common-acoustic-poles/zeros (CAPZ) approximation is a more efficient way to model head-related transfer functions (HRTF's). In CAPZ approximation, a group of HRTF's can share a set of poles but use their own zeros. Previous CAPZ works, such as the Prony, Shanks and iterative prefiltering methods, were all based on the linearized least-square criterion. A new state-space approach, jointly balanced model truncation, is proposed by using singular value decomposition of a joint Hankel matrix. The proposed approach can choose the suitable order of IIR filters for HRTF's approximation according to the distribution of singular values but previous works can't. The proposed method is also modified to permit different orders for pole and zero. Computer simulations of these approaches are included for comparison.

1. INTRODUCTION

Head-related transfer functions (HRTF's), which convey sound transmission characteristics from different spatial locations to both ears, play an important role in 3-D sound processing. In order to completely describe channels' characteristics around the head, many HRTF's have to be measured and stored, which makes real-time implementation difficult. Approximation of FIR by IIR filters can reduce such an enormous data set.

To further save processing time and the memory size, we can model a group of HRTF's using a set of common poles but individual zeros, which is called common-acoustic-poles and zeros (CAPZ) approximation. Compared to conventional pole/zero models, CAPZ approximation is more efficient because it needs fewer poles in respect of a group of HRTF's. Three previous methods developed for CAPZ modeling all use the least-square criterion to minimize approximation error [3][4].

Based on balanced model truncation (BMT)[1], a new approach, jointly balanced model truncation (jointly BMT), is proposed. Using singular value decomposition, the proposed approach extracts the most significant singular vectors to effectively model a group of HRTF's. To further improve jointly BMT, we use iterative prefiltering to determine zero coefficients with a different order from that of pole.

2. COMMON-ACOUSTIC-POLES/ZEROS APPROXIMATION

Each HRTF can be viewed as an long-duration FIR filter which can be approximated by using an IIR filter. Given a group of N HRTF's denoted as $F_1(z), F_2(z), ..., F_N(z)$. In CAPZ approximation, we try to model these HRTF's by the formulation

$$\widehat{F}_i(z) = \frac{Q_i(z)}{P(z)} = \frac{b_{i0} + b_{i1}z^{-1}+, ..., +b_{iq}z^{-q}}{1 + a_1 z^{-1} + a_2 z^{-2}+, ..., +a_p z^{-p}}$$

where $\{1, a_1, a_2, ..., a_p\}$ is common-pole coefficients and $\{b_{i0}, b_{i1}, .., b_{iq}\}$ is individual-zero coefficients, which means a group of approximated HRTF's can share a set of poles but possess their own zeros. To determine $P(z)$ and $Q_i(z)$, it is the most direct way to minimize group error

$$\sum_{i=1}^{N} \|E_i(z)\|^2$$

where $E_i(z) = F_i(z) - \widehat{F}_i(z)$. It is very difficult to solve these nonlinear equations, however. In the following, we will discuss 3 linearized least-square approaches and the state-space model.

2.1. Prony, Shanks and Iterative prefiltering methods

The Prony, Shanks and iterative prefiltering methods have something to do with one another. We just mention their connection here. The details for these three methods can be found in [3][4]. As a compromise, the Prony method did not directly minimize the group error but the group filtered error

$$\sum_{i=1}^{N} \left\| \widehat{E}_i(z) \right\|^2$$

where $\widehat{E}_i(z) = P(z)E_i(z)$. In the time domain, minimizing group filtered error is just a least-square problem and common-pole and individual-zero coefficients can be determined. The Shanks method determined $P(z)$ using the same way as the Prony method did. The difference is that the Shanks method directly minimized group error to determine $Q_i(z)$ because our initial goal, minimizing group error, is no longer a nonlinear problem after $P(z)$ is determined. It is obvious that the Shanks method for sure has better performance than the Prony method does because the Shanks method directly minimized group error while determining $Q_i(z)$. The Prony and Shanks methods are basically based on the minimization of filtered error, so they have the problem that direct error is not suppressed enough. The iterative prefiltering method, which can be seen as extension of the Shanks method, iteratively minimized

$$E^{(i+1)}(z) = \frac{P^{(i+1)}(z)F(z) - Q_i^{(i+1)}(z)}{P^{(i)}(z)}$$

to reach an optimum solution. $P^{(i+1)}(z)$ and $Q_i^{(i+1)}(z)$ can be determined by minimizing $E^{(i+1)}(z)$ when $P^{(i)}(z)$ is fixed. In a few iterations, the group error will converge. The initial guess of denominator coefficients can be found by the Prony method.

2.2. Jointly Balanced Model Truncation

We have to briefly outline BMT here, because jointly BMT is derived from BMT. The details for BMT can be found in [2]. We start with an FIR filter $F(z)$ with order n written as:

$$F(z) = f_0 + f_1 z^{-1} + \cdots + f_n z^{-n}.$$

In the state-space model, $F(z)$ can be expressed as a set of difference equations:

$$\begin{aligned} x(r+1) &= Ax(r) + Bu(r) \\ y(r) &= Cx(r) + Du(r) \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots\vdots & \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C = [f_1, f_2, \ldots, f_n], \quad D = f_0.$$

The transfer function $F(z)$ is related to the state-space model by the formulation $F(z) = C(zI - A)^{-1}B + D$.

$F(z)$ can be approximated by the pth order reduced balanced system $(A^{(p)}B^{(p)}C^{(p)}D)$ using BMT. A low-order IIR filter $F^{(p)}(z)$ is related to $(A^{(p)}B^{(p)}C^{(p)}D)$ by

$$F^{(p)}(z) = C^{(p)}(zI - A^{(p)})^{-1}B^{(p)} + D \tag{1}$$

where

$$A^{(p)} = V_p^T A V_p, B^{(p)} = V_p^T B, C^{(p)} = CV_p, D = f_0 \tag{2}$$

and p indicates the pole order which equals the zero order. To determine V_p, we can define a Hankel matrix as follows:

$$H = \begin{bmatrix} f_1 & f_2 & \ldots & f_n \\ f_2 & f_3 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f_n & 0 & \cdots & 0 \end{bmatrix}. \tag{3}$$

Because H is a symmetric matrix, it can be factorized as:

$$H = V \Lambda V^T. \tag{4}$$

V_p is an $n \times p$ matrix obtained from the following partition:

$$V = [V_p\ V_{n-p}]. \tag{5}$$

As we mentioned before, BMT can only be used to decide the approximated IIR filter for individual HRTF's. Based on BMT, jointly BMT is proposed to approximate a group of HRTF's using common-poles IIR filters. Equation (1) indicates that the poles of an approximated IIR filter depend on $A^{(p)}$ which is determined by V_p from (2). From above, we learn that a group of HRTF's can be approximated by using common poles and individual zeros if Hankel matrices belonging to different HRTF's share a set of truncated eigenvectors V_p. In the following, we try to find V which can be shared by a group of Hankel matrices because V_p is given by (5).

In finding eigenvectors of H , we try to find $\mathbf{x_1}$ which can satisfy:

$$\max_{||\mathbf{x_1}||=1} ||H \cdot \mathbf{x_1}||^2$$

where $\mathbf{x_1}$ is an eigenvector belonging to the largest eigenvalue of H. In turn, we can find $\mathbf{x_2}, ..., \mathbf{x_n}$ which are orthogonal eigenvectors of H. When given a group of Hankel matrices $H_1, H_2, ..., H_N$, we also try to find an eigenvector $\mathbf{x_1}$ which can satisfy:

$$\max_{||\mathbf{x_1}||=1} ||H_1 \cdot \mathbf{x_1}||^2, \max_{||\mathbf{x_1}||=1} ||H_2 \cdot \mathbf{x_1}||^2, ..., \max_{||\mathbf{x_1}||=1} ||H_N \cdot \mathbf{x_1}||^2.$$

This attempt is not likely to succeed unless $H_1, H_2, ..., H_N$ have a common set of eigenvectors. As a compromise, we try to find $\mathbf{x_1}$ which can satisfy:

$$\max_{||\mathbf{x_1}||=1} \left(||H_1 \cdot \mathbf{x_1}||^2 + ||H_2 \cdot \mathbf{x_1}||^2 + ... + ||H_N \cdot \mathbf{x_1}||^2\right)$$

$$= \max_{||\mathbf{x_1}||=1} \left\| \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_N \end{bmatrix} \cdot \mathbf{x_1} \right\|^2$$

which means $\mathbf{x_1}$ is the eigenvector belonging to the largest eigenvalue of $\mathcal{H}^T\mathcal{H}$ where the joint Hankel matrix $\mathcal{H}$ is written as:

$$\mathcal{H} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_N \end{bmatrix}. \tag{6}$$

In CAPZ modeling, there are a group of N HRTF's denoted as $F_1(z), F_2(z), \ldots, F_N(z)$. For each HRTF, we can define a Hankel matrix H_i as (3). In order to find a set of eigenvectors shared by all Hankel matrices, we can create a joint Hankel matrix $\mathcal{H}$ by cascading $H_1, H_2, \ldots, H_N$ as (6). We assume $\mathcal{H}$ is an $m \times n$ matrix (m>n) which can be factorized as follows:

$$\mathcal{H} = U\Lambda V^T \tag{7}$$

where V contains the eigenvectors of $\mathcal{H}^T\mathcal{H}$. Finally, each $F_i(z)$ can be approximated by the pth order reduced system $(A^{(p)}B^{(p)}C_i^{(p)}D_i)$ where

$$A^{(p)} = V_p^T A V_p, B^{(p)} = V_p^T B, C_i^{(p)} = C_i V_p, D_i = f_{i0} \tag{8}$$

$i = 1, 2, \ldots, N$ and V_p is given by (5). The choice of order p depends on the distribution of Σ's diagonal elements. The substantially smaller part of Σ's diagonal elements can be viewed as redundancy which has little effect on the system response. We can convert the pth order reduced system $(A^{(p)}B^{(p)}C_i^{(p)}D_i)$ into the transfer function using $F_i^{(p)}(z) = C_i^{(p)}(zI - A^{(p)})^{-1}B^{(p)} + D_i$ where $F_i^{(p)}(z)$ is the approximated IIR filter of $F_i(z), i = 1, 2, \ldots, N$.

2.3. Modified Jointly Balanced Model truncation

For CAPZ approximation, the desired IIR filters derived from jointly BMT must have the same orders in numerator and denomerator ($p = q$). Three previous methods do not have this kind of restriction. To make jointly BMT more flexible ($p \neq q$), we try to determine zero coefficients of jointly BMT using iterative prefiltering. That is, we use jointly BMT to decide the common poles which are used as the initial value for iterative prefiltering. Originally, the initial guess of denomerator coefficients for iterative prefiltering was solved from the Prony method and within a few iterations iterative prefiltering would converge. The proposed method, modified jointly balanced model truncation (modified JBMT), has comparable performance as iterative prefiltering does but it can converge in one iteration.

3. SIMULATION RESULTS

The HRTF's data we use here are measured by MIT[5]. For CAPZ modeling, we choose 14 HRTF's measured for the left ear as a group with the azimuth=0 degree and elevations ranging from -40 degree to 90 degree by 10-degree increase. Before simulation, these HRTF's were prefiltered. Initial time delays were removed and HRTF's with longer data length were cut from the tail to make them have same data length. For performance comparison, we define a group error index as:

$$\sqrt{\frac{\left\|F_1 - F_1^{(p)}\right\|^2 + \left\|F_2 - F_2^{(p)}\right\|^2 + ... + \left\|F_N - F_N^{(p)}\right\|^2}{\|F_1\|^2 + \|F_2\|^2 + ... + \|F_N\|^2}}$$

where F_i is the actual impulse response and $F_i^{(p)}$ is the approximated HRTF.

Starting with Fig. 1, we plot the diagonal elements of Σ versus order p of reduced systems. Fig. 1 reveals that the more sharply the curve falls to zero, the more efficiently and accurately we can model a group of HRTF's. That is, when singular values are negligible, singular vectors belonging to them are not necessarily engaged in as (5). In Fig. 2, we plot the group error index versus order p using the jointly BMT method. The result shows that at the beginning, the group error index drops sharply as p increases but it decreases more slowly when p is bigger than 15 , which fit in with the result in Fig. 1. These two figures show us how to decide the oder p of reduced systems. Previous works can't offer us this kind of help.

We compare conventional pole/zero modeling and CAPZ approximation in Table 1. We also compare the group error index for four methods under the condition that the pole number (=zero number) of desired IIR filters equals 12 . From Table 1, it is clear that conventional pole/zero modeling can model a group of HRTF's more efficiently but all IIR filters have to use their own poles. CAPZ approximation sacrifices some accuracy so that it can save computational cost because all approximated HRTF's in a group share a set of poles. In terms of different approaches, iterative prefiltering is the best one and jointly BMT follows.

In Fig. 3, we compare CAPZ models derived from the Prony, Shanks, iterative prefiltering and modified JBMT methods. For each approach, we plot a curve by changing the zero number of desired IIR filters when the pole number equals 5. The result shows that the proposed method is comparable with the iterative prefiltering method but outperforms the Prony and Shanks methods.

4. CONCLUSION

In real-time applications, processing time is crucial. We propose a new approach, jointly BMT, for CAPZ approximation which can efficiently model a group of HRTF's. To further improve the proposed method, we extend jointly BMT to modified JBMT such that we can use different oders for pole and zero of desired IIR filters. Simulation results show that the proposed methods have better performance than the previous Prony and Shanks methods.

5. REFERENCES

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Table 1. Comparison of group error indices for CAPZ approximation and conventional models which use individual poles and individual zeros.

	$\{\frac{Q_i(z)}{P_i(z)}\}$	$\{\frac{Q_i(z)}{P(z)}\}$
Prony	0.2229	0.4063
Shanks	0.2052	0.3042
Iterative prefiltering	0.1243	0.2115
Jointly BMT	0.1616	0.2197

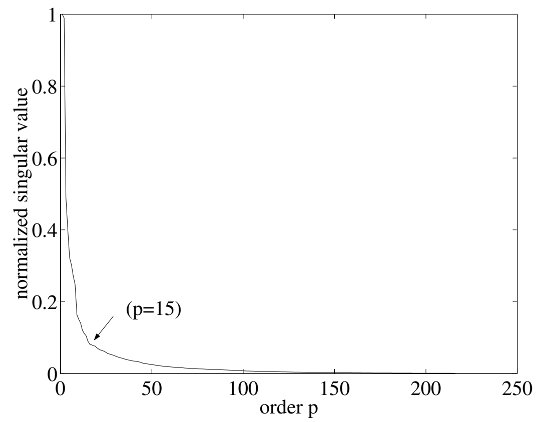

Fig. 1. Singular values plot versus the order of desired IIR filters

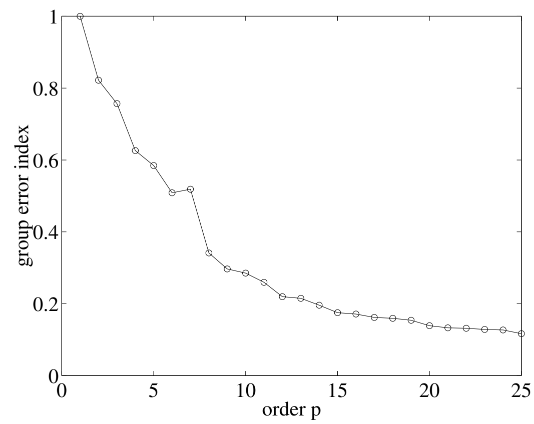

Fig. 2. Group error indices plot versus the order of desired IIR filters using jointly BMT

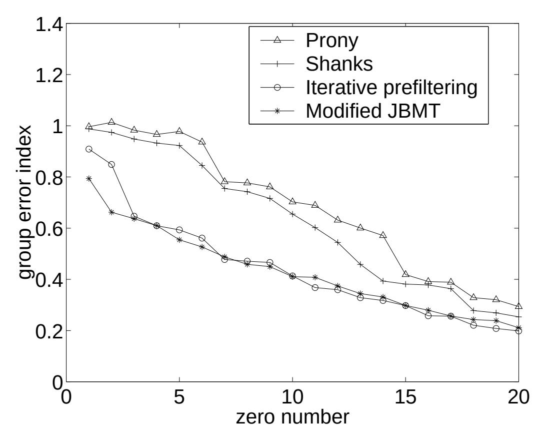

Fig. 3. Group error indices comparison of four CAPZ models (pole number = 5).